October 14

Get Whiteboards

Finish VPython simulation of a spacecraft flyby

Ponderable:

Jack and Jill are maneuvering a 3000 kg boat near a dock. Initially the boat's position is <2, 0, 3> m and its speed is 1.3 m/s. As the boat moves to position <4, 0, 2> m, Jack exerts a force <-400, 0, 200> N and Jill exerts a force <150, 0, 300> N.

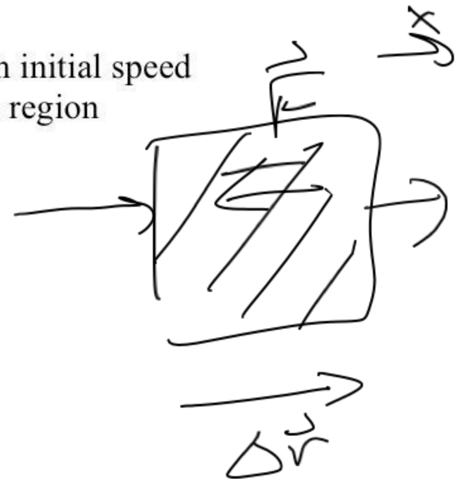
$$\Delta E_{\text{sys}} = W_{\text{surr}}$$
$$\left(m_{\text{boat}}c^2 + K\right)_{\text{final}} - \left(m_{\text{boat}}c^2 + K\right)_{\text{initial}} = W_{\text{Jack}} + W_{\text{Jill}}$$
$$K_f = K_i + W_{\text{Jack}}$$
$$\tfrac{1}{2} m_{\text{boat}} v_f^2 = \tfrac{1}{2} m_{\text{boat}} v_i^2 + \vec{F}_{\text{Jack}} \bullet \Delta\vec{r}$$
$$v_f^2 = \frac{2}{m_{\text{boat}}}\left(\tfrac{1}{2} m_{\text{boat}} v_i^2 + \vec{F}_{\text{Jack}} \bullet \Delta\vec{r}\right)$$
$$v_f = \sqrt{\frac{2}{3000\ \text{kg}}\left(\tfrac{1}{2}\left(3000\ \text{kg}\right)\left(1.3\,\tfrac{\text{m}}{\text{s}}\right)^2 - 1000\ \text{J}\right)}$$
$$v_f = \sqrt{\frac{2}{3000\ \text{kg}}\left(\tfrac{1}{2}\left(3000\ \text{kg}\right)\left(1.3\,\tfrac{\text{m}}{\text{s}}\right)^2 - 1000\ \text{J}\right)}$$
$$v_f = \sqrt{\frac{2}{3000\ \text{kg}}\left(2535\ \text{J} - 1000\ \text{J}\right)} = \sqrt{\frac{2}{3000\ \text{kg}}\left(1535\ \text{J}\right)}$$
$$v_f = 1.0\,\tfrac{\text{m}}{\text{s}}$$

Ponderable:

An electron traveling at a speed 0.99c encounters a region where there is a constant electric force directed opposite to its momentum. After traveling 3 m in this region, the electron's speed was observed to decrease to 0.93c. What was the magnitude of the electric force acting on the electron?

$\Delta E = W$

System: the electron
Surroundings: the region with the field
Initial state: electron at initial position, with initial speed
Final state: after electron moved 3 m in the region



$$\Delta E_{\text{Sys}} = W_{\text{Surr}}$$

$$E_f - E_i = \vec{F}_{\text{net}} \bullet \Delta\vec{r} = -F\Delta x$$

$$\left(\frac{1}{\sqrt{1-0.93^2}} m_{\text{electron}} c^2\right) - \left(\frac{1}{\sqrt{1-0.99^2}} m_{\text{electron}} c^2\right) = -F\Delta x$$

$$(2.72)\left(8.2\times10^{-14}\ \text{J}\right) - (7.09)\left(8.2\times10^{-14}\ \text{J}\right) = -F\Delta x$$

$$-3.6\times10^{-13}\ \text{J} = -F\Delta x$$

$$F = \frac{3.6\times10^{-13}\ \text{J}^2}{3\ \text{m}} = 1.19\times10^{-13}\ \text{N}$$

non-relativistic (wrong):

$$\Delta E_{\text{Sys}} = W_{\text{Surr}}$$

$$\left(m_{\text{electron}} c^2 + K\right)_{\text{final}} - \left(m_{\text{electron}} c^2 + K\right)_{\text{initial}} = W_{\text{Surr}}$$

$$K_f - K_i = \vec{F}_{\text{net}} \bullet \Delta\vec{r} = -F\Delta x$$

$$\tfrac{1}{2} m_{\text{electron}} v_f^2 - \tfrac{1}{2} m_{\text{electron}} v_i^2 = -F\Delta x$$

$$\tfrac{1}{2}\left(9.1\times10^{-31}\ \text{kg}\right)\left(0.93\times3\times10^8\right)^2 - \tfrac{1}{2}\left(9.1\times10^{-31}\ \text{kg}\right)\left(0.99\times3\times10^8\right)^2 = -F\Delta x$$

$$3.5\times10^{-14}\ \text{J} - 4.0\times10^{-14}\ \text{J} = -F\Delta x$$

$$F = \frac{0.5\times10^{-14}\ \text{J}}{3\ \text{m}} = 1.57\times10^{-15}\ \text{N}$$

$$\vec{F}\cdot\Delta\vec{r} = F_x\Delta x + F_y\Delta y + F_z\Delta z$$
$$= |\vec{F}||\Delta\vec{r}|\cos\theta$$

A new unit of energy

$$1\,eV = 1.6 \times 10^{-19}\,J$$

$$M = 10^6$$

E when $v = 0.93c$

$$E = \frac{1}{\sqrt{1 - 0.93^2}}\, m_e c^2 = 2.72\, m_e c^2$$

$$= (2.72)(9.1 \times 10^{-31}\,kg)(3 \times 10^8\,m/s)^2 = 2.2 \times 10^{-13}\,J$$

$$= 2.2 \times 10^{-13}\,J\ \frac{1\,eV}{1.6 \times 10^{-19}\,J} = 1.4e6\,eV$$

$$= 1.4\,MeV$$

Comparison of energy and momentum definitions

Odd tables: Write definition of energy and a graph showing total energy vs. speed.

$E = \gamma m c^2$

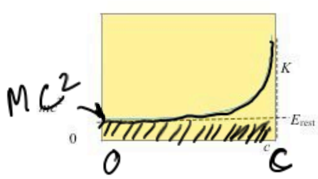
Even tables: Write definition of momentum and a graph showing magnitude of momentum vs. speed.

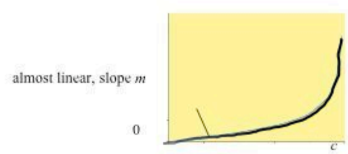
$\vec{p} = \gamma m \vec{v}$

$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$

Comparison of energy and momentum definitions

$$E = \gamma mc^2 = \frac{1}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}} mc^2 \qquad \vec{p} = \gamma m\vec{v} = \frac{1}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}} m\vec{v}$$





Comparison of Energy and Momentum Principles

Odd tables: Write down Momentum Principle

Even tables: Write down Energy Principle

Comparison of Energy and Momentum Principles

$$\vec{p}_f = \vec{p}_i + \vec{F}_{\text{net}}\Delta t \qquad E_f = E_i + W = E_i + \vec{F}_{\text{net}} \bullet \Delta\vec{r}$$

principles where physics is
describe how change in time/space

$$\left.\begin{array}{l} E = \gamma m c^2 \\ \vec{p} = \gamma m \vec{v} \end{array}\right\} \text{definitions}$$

$$\vec{\xi} = \frac{M^2 \vec{v}}{(|\vec{v}|)^3 + 1}$$

Connecting energy and momentum

$$E = \gamma m c^2 \qquad \vec{p} = \gamma m \vec{v}$$

$$= \frac{mc^2}{\sqrt{1-(v/c)^2}} \qquad = \frac{m\vec{v}}{\sqrt{1-(v/c)^2}}$$

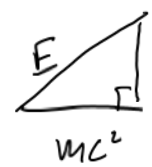
$$E^2 = \frac{m^2c^4}{1-(v/c)^2}$$

$$|\vec{p}|^2 = \frac{m^2v^2}{1-(v/c)^2}$$

invariant

$$E^2 - (pc)^2 = (mc^2)^2$$

indep of ref frame



Energy Principle for a system

$$\Delta E = W$$

Change in particle energy

= Work done on particle

Change from particle to system

$$\Delta E_{sys} = W_{surroundings} \Rightarrow E_{sys,f} = E_{sys,i} + W_{surr.}$$

Things can change in system

If system contains more than one particle there is "interaction" or "potential" energy U

$$\Delta\left(E_{particles} + U_{pairs\ of\ particles}\right) = W_{by\ external\ forces}$$

Changing mass

$$E_f = E_i + W$$

$$mc^2 + K_f = mc^2 + K_i + W$$

This assumes mass does not change

What if mass changes?

Neutron at rest decays to proton, electron and antineutrino. What is K_{total} of final decay products?

$m_n = 939.6\ MeV/c^2$ $\quad E_i = m_n c^2$

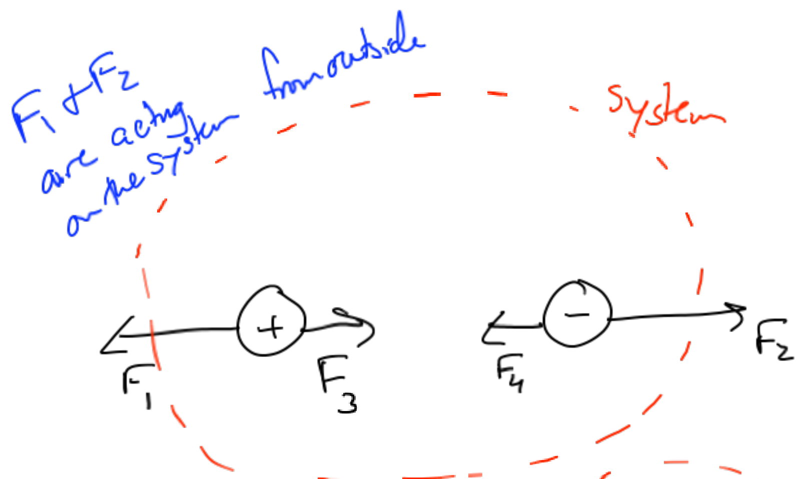
$m_p = 938.3\ MeV/c^2$ $\quad E_f = m_p c^2 + K_p + m_e c^2 + K_e + K_\nu$

$m_e = 0.511\ MeV/c^2$

$m_\nu \approx 0$

$E_f = E_i + W = E_i$

$K_p + K_e + K_\nu = m_n c^2 - m_p c^2 - m_e c^2 = 0.8\ MeV$



$$\Delta(E_1 + E_2) = W_1 + W_2 + W_3 + W_4$$

bring everything internal to system to LHS, everything external to RHS

$$\Delta(E_1 + E_2) - W_3 - W_4 = W_1 + W_2$$

$$\Delta U = -(W_3 + W_4)$$ change in potential energy

$$\Delta(E_1 + E_2 + U) = W_1 + W_2$$ or $$\Delta E_{system} + \Delta U_{sys} = W_{surroundings}$$

Relating force to potential energy

$$\Delta U = -W_{internal}$$

$$= -\vec{F} \cdot \Delta \vec{r} \quad \text{internal interactions}$$

$$-F = \frac{\Delta U}{\Delta r} \Rightarrow F = \frac{-dU}{dr}$$